

Acceptance speech

20 June 2024

Claire Voisin, awardee in the Basic Sciences category (16th edition)

It is a great honor for me to share with Yakov Eliashberg the Frontiers of Knowledge Award in the category of Basic Sciences. I find it remarkable that the BBVA Foundation generously supports arts and science, without prejudging their outcome or applications.

In these difficult times, it would be totally unrealistic to claim that mathematicians doing research in the field of pure mathematics work for the welfare of humanity. Nevertheless, I do believe that mathematical knowledge is essential to knowledge and intellectual life in general, because it is built on the universal principle that every statement has to be proved in order to be considered true. The question of what mathematics is useful for is different, and is a matter of personal taste and appreciation. I think of the Italian Jesuit Matteo Ricci, who lived in China for 30 years at the end of the 16th century, and tried to convert Chinese people to Christianity using reasoning and logical arguments. He was not very successful in converting people but he compiled a Portuguese-Chinese dictionary, and translated the first 6 volumes of Euclid elements from Latin to Chinese, and the Confucian classics from Chinese to Latin. This created a beautiful bridge between Chinese and occidental cultures, made of mathematics and philosophy.

Mathematical theories and objects appear through an abstract extrapolation of the real world. After a long detour that requires audacity, freedom and creativity, they help to understand, describe, and interpret our physical and intuitive experience. This is the case of Riemannian geometry, which is the general study of metric spaces and their curvature, like the surface of the Earth. On a Riemannian manifold, one can measure the norm of a tangent vector, hence the length of trajectories. Likewise, symplectic geometry, which studies spaces on which one can compute the area of the parallelogram generated by two tangent vectors, and its companion called "contact geometry" appeared originally in mechanics, and the great names of Hamilton, Lagrange and Legendre now appear everywhere in symplectic and contact geometry.

Complex geometry, which can be described as analysis of the complex numbers, is an analytic development of the study of polynomial functions in several complex variables. In real dimension 2, it is very close to Riemannian geometry, but in higher dimension, complex geometry is nicely related to Riemannian geometry only if the considered complex manifolds are Kähler. This contains the case of complex projective manifolds, which are studied in complex algebraic geometry. In the Kähler case, the manifolds also admit a symplectic structure given by the Kähler form. This creates a link between complex geometry and symplectic geometry that has been generalized by Gromov, who introduced almost complex structures in symplectic geometry and pseudoholomorphic curves.

Yakov Eliashberg worked on the topology of Stein manifolds, defined by holomorphic equations in complex affine space. In particular, he gave a topological characterization of almost complex manifolds which deform to Stein manifolds. His pioneering ideas made him a leader both in symplectic and contact geometry.

An important part of my research has been devoted to the topology of projective complex manifolds. I proved for example that it is more restricted than the topology of compact Kähler manifolds. Recently, I have been working with Debarre, Huybrechts, and Macri on hyper-Kähler manifolds, a subject introduced by Hitchin and Beauville in the 1980s. They are complex manifolds that admit many symplectic structures, in particular Kähler and holomorphic, that are exchanged via a twistor space construction. We obtained a topological characterization of a certain class of such objects in dimension 4.

Today I am happy to celebrate in Spain the beauty of complex geometry and symplectic topology. I salute my colleagues and friends from Barcelona, Bilbao and Madrid, and I heartily thank the BBVA Foundation for this award, which is a great recognition and a strong encouragement for my research.